

An advanced precision analysis of the SM vacuum stability

A. V. Bednyakov^{a,b,1}

^a Joint Institute for Nuclear Research, 141980, Dubna, Russia

^b Dubna State University, 141982, Dubna, Russia

Доклад посвящен проблеме стабильности электрослабого вакуума в Стандартной модели фундаментальных взаимодействий. В качестве инструмента исследования рассматривается эффективный потенциал поля Хиггса, который при учете радиационных поправок может обладать дополнительным, более глубоким минимумом. Обсуждаются различные методы и приближения, используемые для вычисления эффективного потенциала. Особое внимание уделяется ренормгрупповому подходу, позволившему провести анализ стабильности на трехпетлевом уровне точностью. С помощью явно калибровочно-инвариантной процедуры находятся ограничения на наблюдаемые массы бозона Хиггса и топ-кварка, совместные с требованием абсолютной стабильности СМ. Демонстрируется важность учета высших поправок теории возмущений. Также обсуждается потенциальная метастабильность СМ и модификации анализа при учете Новой физики.

The talk is devoted to the problem of stability of the Standard Model vacuum. The effective potential for the Higgs field, which can potentially exhibit additional, deeper minimum, is considered as a convenient tool for addressing the problem. Different methods and approximations used to calculate the potential are considered. Special attention is paid to the renormalization-group approach that allows one to carry out three-loop analysis of the problem. By means of an explicit gauge-independent procedure the absolute stability bounds on the observed Higgs and top-quark masses are derived. The importance of high-order corrections is demonstrated. In addition, potential metastability of the SM is discussed together with modifications of the analysis due to some New Physics.

The Standard Model, although being established as a quantum-field theory in mid 70s of the last century, turns out to be a perfect description of many phenomena at scales accessible to current accelerator experiments. The Lagrangian of the model

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Gauge}}(g_1, g_2, g_s) + \mathcal{L}_{\text{Yukawa}}(Y_u, Y_d, Y_l) + \mathcal{L}_{\text{Higgs}}(\lambda, m^2) \quad (1)$$

depends on the gauge couplings g_s, g_2, g_1 , which sets the strength of $SU(3) \times SU(2) \times U(1)$ gauge interactions, (matrix) Yukawa couplings $Y_{u(d)}$ and Y_l for the up(down)-type quarks and leptons coupled to the Higgs doublet Φ , and the parameters of the tree-level Higgs potential

$$V(\Phi) = m^2 |\Phi|^2 + \lambda |\Phi|^4, \quad \Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(\phi + i\chi) \end{pmatrix}. \quad (2)$$

¹E-mail: bednya@theor.jinr.ru

Talk given at the International Session-Conference of SNP PSD RAS “Physics of Fundamental Interactions”, JINR, Dubna, 2016

For $m^2 < 0$ the potential has degenerate minima, which are characterized by non-zero Higgs field vacuum expectation value (vev) $|\langle\Phi\rangle|^2 \equiv v^2/2 \neq 0$. As usual it is a signal of the spontaneous symmetry breaking. Due to the degeneracy one can choose the vacuum, for which only ϕ component has non-zero vev $\langle\phi\rangle = v$. The latter can be expressed in terms of the Lagrangian parameters (in the leading order) as a space-time independent solution of the classical equation of motion

$$\left. \frac{\partial V(\phi)}{\partial \phi} \right|_{\phi=v} = 0 \Rightarrow v^2 = -\frac{m^2}{\lambda} \quad (3)$$

Due to interactions with the Higgs-field condensate $\langle\phi\rangle$, elementary particles acquire masses proportional to the corresponding coupling. It is this fact that allows one, given the SM relation $M_h^2 = 2\lambda v^2$, to deduce the value of the self-coupling λ from the Higgs mass $M_h^{\text{exp}} = 125.09(24)$ GeV, measured at the LHC.

The obtained value turns out to be rather special. Before the discovery of the Higgs boson both theorist and experimentalist were eagerly searching for hints of New Physics (NP). One way to address this issue is to analyze self-consistency of the SM by studying high-energy behavior of the relevant effective couplings.

In Fig. 1 quite an old plot [1] with two curves is presented in the plane (M_h, Λ) . The upper (red) line corresponds to the “triviality” constraint - at the corresponding scale Λ the self-coupling hits a Landau pole/becomes non-perturbative. It is easy to see that the observed value of the Higgs mass lies significantly below this line in the whole range of Λ .

The second (green) curve represents a lower bound on the mass originating from the fact that the Higgs potential (2) with scale-dependent λ becomes unstable for a given scale Λ , leading to possible decay of the SM vacuum. The curve looks more interesting since M_h^{exp} lies in a dangerous vicinity of the line. To decide whether the SM can be extrapolated self-consistently up to the Planck scale or New Physics should be introduced to cope with the above-mentioned instability one has to perform an elaborated analysis taking into account various radiative corrections.

A proper way to study symmetry breaking in the SM is to consider effective potential $V_{\text{eff}}(\phi)$, which also takes into account contribution ΔV from vacuum fluctuations of all fields of the model

$$V_{\text{eff}}(\phi) = V(\phi) + \Delta V(\phi) \equiv m_{\text{eff}}^2(\phi) \frac{\phi^2}{2} + \lambda_{\text{eff}}(\phi) \frac{\phi^4}{4}, \quad \Delta V(\phi) = \sum_n \hbar^n \Delta^{(n)} V(\phi), \quad (4)$$

where m_{eff} and λ_{eff} are introduced. For ΔV loop expansion is implied [2, 3]. In the

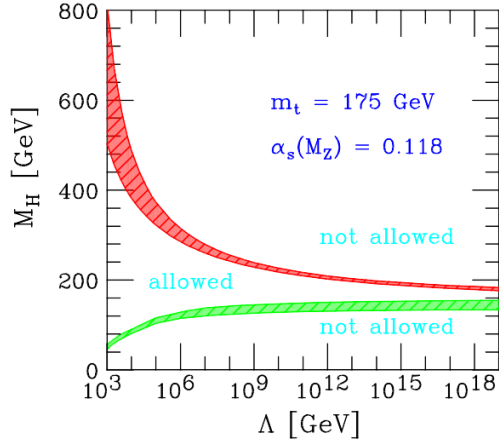


Fig. 1: Triviality (upper) and stability (lower) bounds on the Higgs mass M_h depending on the New Physics scale Λ [1]

Particle	κ	κ'	n
W^\pm	$g_2^2/4$	0	2×3
Z	$(g_2^2 + g_1^2)/4$	0	3
t	$y_t^2/2$	0	4×3
h	3λ	m^2	1
G	λ	m^2	3×1

Table 1: Field-dependent masses of the SM particles $M^2(\phi) = \kappa\phi^2 + \kappa'$ together with the corresponding numbers of degrees of freedom n . Massive vector bosons W^+ , W^- , and Z , top-quark t , Higgs boson h and Goldstone bosons $G = G^{\pm,0}$ are considered.

Landau gauge we have¹

$$\Delta^{(1)}V(\phi) = \int \frac{d^4 k}{2(2\pi)^4} \text{STr} \ln(k^2 + M^2(\phi)), \quad (5)$$

where field-dependent particle masses $M^2(\phi) = \kappa\phi^2 + \kappa'$ are introduced (see Table 1). The supertrace STr counts positively (negatively) bosonic (fermionic) degrees of freedom n . From (5) one can clearly see the origin of instability [4] – the fermionic contribution drives the potential to negative values. High-order terms in the expansion (4) can be found from vacuum graphs involving $M^2(\phi)$ in propagators.

Before going further, let us mention an annoying subtlety of gauge-dependence of the effective potential [3] at general values of ϕ . It manifests itself as a dependence on auxiliary gauge-fixing parameters ξ_i . The latter is controlled by the following Nielsen identity [5]:

$$\xi \frac{\partial V_{\text{eff}}}{\partial \xi} = -C(\phi, \xi) \frac{\partial V_{\text{eff}}}{\partial \phi}, \quad (6)$$

which tells us that the change due to ξ variation in V_{eff} can be compensated by appropriate field rescaling. It turns out that only at extrema¹ the corresponding energy-density is gauge-invariant. We should keep this in mind, when discussing the stability issue of the SM.

In order to address the problem of the SM stability, we consider how the values of V_{eff} at *extrema* are related to each other. Denoting higgs field value at possible additional minimum as v' , we distinguish the case of absolute stability of the EW vacuum, $V_{\text{eff}}(v) < V_{\text{eff}}(v')$. When $V_{\text{eff}}(v) = V_{\text{eff}}(v')$ we have *critical* situation with degenerate minima

$$\left. \frac{\partial V_{\text{eff}}}{\partial \phi} \right|_{\phi=v} = \left. \frac{\partial V_{\text{eff}}}{\partial \phi} \right|_{\phi=v'} = 0, \quad V_{\text{eff}}(v) = V_{\text{eff}}(v'). \quad (7)$$

Finally, for $V_{\text{eff}}(v) > V_{\text{eff}}(v')$ our ground state turns out to be unstable. The latter case deserves further study, since there exist a possibility for our Universe to have negligible probability to decay into the true vacuum (“metastability”). In what follows we mostly discuss absolute stability and just give a brief comment on potential metastability.

¹The integral is obviously divergent and can be defined within some renormalization scheme.

¹Having in mind that $V'_{\text{eff}}(\phi) \equiv J_\phi$, the external source J_ϕ vanishes at extrema.

It turns out that for large field values $\phi \gg v$ one can not use truncated perturbation-theory (PT) series and has to reorganize (improve) the expansion. In a nutshell, the problem stems from the fact that beyond the tree-level approximation terms of the following form

$$a^{L+1}(\mu) \left[\ln \frac{\phi^2}{\mu^2} \right]^\kappa, \quad \kappa = 1, \dots, L \quad (8)$$

will arise in L -loop contribution to V_{eff} . In (8) μ corresponds to some normalization scale¹, at which a set of scale-dependent (running) couplings $a(\mu) = \{a_i(\mu)\}$,

$$(4\pi)^2 a_i = \{g_1^2, g_2^2, g_s^2, y_b^2, y_t^2, \lambda\}, \quad (9)$$

is defined. The truncated finite-order result (4) has residual dependence on μ and, obviously, has limited precision for $\phi \gg \mu$. A well-known solution to this kind of problems is re-summation by means of renormalization group (RG), which, roughly speaking, corresponds to the choice $\mu = \phi$.

Further simplification can be achieved, when for large field values the full effective potential is approximated by the tree-level $\lambda\phi^4$ term

$$V_{\text{eff}}(\phi) \xrightarrow{\phi \gg v} \lambda_{\text{eff}}(\phi) \frac{\phi^4}{4} \simeq \lambda(\mu = \phi) \frac{\phi^4}{4}, \quad (10)$$

with running $\lambda(\mu = \phi)$ in $\overline{\text{MS}}$ scheme [6]. In the latter case *only* leading logarithmic (LL) contribution ($\kappa = L$) from (8) can be re-summed consistently. Applying the critical conditions (7) to this simple case one obtains [7]

$$\lambda(\mu) = 0, \quad \beta_\lambda(\mu) = 0 \quad (11)$$

with $(4\pi)^2 \beta_\lambda \equiv d\lambda/d\ln\mu^2$ corresponding to the beta-function of λ (see below). As independent variables in (11) one usually chooses μ and a physical mass of interest, either M_h or M_t . Keeping all other parameters fixed, critical scale μ^{crit} and critical mass M_h^{crit} (M_t^{crit}) are deduced. From (5) it is easy to convince oneself that the obtained M_t^{crit} corresponds to the upper bound on the physical mass, while M_h^{crit} — to the lower bound.

Strictly speaking, if one wants to go beyond the LL approximation, Eq. (7) should be used in place of (11) and non-logarithmic (“finite”) terms in λ_{eff} should be considered. The latter are known at two loops [6,8] within the full SM². This allows one to consistently re-summ next-to-next-to-leading (NNLL) logarithmic terms ($\kappa = L - 2$) by utilizing the system of coupled RG equations at the three-loop order

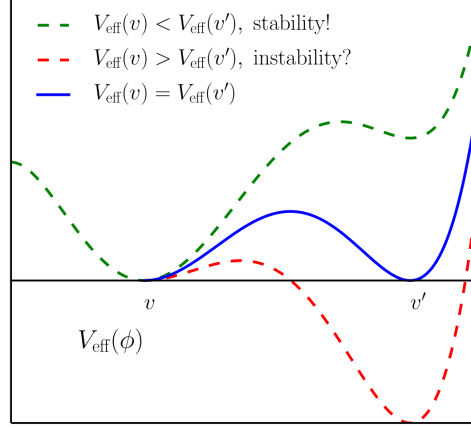


Fig. 2: Different behavior of the effective potential. To decide, whether the EW vacuum with $\langle\phi\rangle = v$ is stable or not, one compares $V_{\text{eff}}(v)$ with $V_{\text{eff}}(v')$.

¹We use $\overline{\text{MS}}$ to define the SM running parameters.

²Partial three- and even four-loop results are also known in Landau gauge [9].

$$\mu^2 \frac{da_i}{d\mu^2} = \beta_{a_i}(a_j), \quad \beta_{a_i} = \beta_{a_i}^{(1)} + \beta_{a_i}^{(2)} + \beta_{a_i}^{(3)} + \dots \quad (12)$$

The appropriate boundary conditions are usually supplied at the EW scale $M_Z \simeq 100$ GeV. Various $\beta_a^{(i)}$ represent i -loop contribution to the beta-function of a and are known for the full SM up to the third order (see [10] and refs. therein). Some leading four-loop corrections to the beta-functions were recently computed in the literature [11].

The boundary values are obtained by the so-called matching procedure, which relate the Lagrangian parameters (in the $\overline{\text{MS}}$ scheme) to a set of (pseudo)observables. It is customary to use the pole masses of the SM bosons, M_W , M_Z , M_h , together with that of fermions M_f to extract (or match) the relevant running parameters at the EW scale. In principle, matching can be done at any scale. However, again, truncated series exhibits bad behavior if μ is chosen far away from a typical scale involved in the definition of the observable, so matching is usually carried out at $\mu \simeq M_Z$ and RG is used for $\mu \gg M_Z$.

Two more constraints are required in order to completely determine the values of the SM parameters in the $\overline{\text{MS}}$ scheme. They usually come from the requirement that the SM should reproduce Fermi theory and QCD with five active flavours, when considered at scales far below the corresponding thresholds (e.g., M_W or M_t). In other words, given the SM one can predict the couplings G_F and $\alpha_s^{(5)}(\mu)$ of the above-mentioned effective theories. At the end of the day, we have the relations of the form (full set can be found, e.g., in Ref. [12])

$$M_h^2 = 2\lambda v^2(1 + \bar{\delta}_h), \quad (4\pi)^2 \alpha_s^{(5)}(\mu) = g_s^2(1 + \bar{\delta}_{\alpha_s}), \quad 2^{1/2} G_F = v^{-2}(1 + \bar{\delta}_r), \quad (13)$$

in which RHS depend only on $\overline{\text{MS}}$ parameters and the renormalization scale μ , while LHS, with the only exception of $\alpha_s^{(5)}(\mu)$, are μ -independent. Various $\bar{\delta}$'s represent radiative corrections and are calculated in PT. As in the case of effective potential, for consistent NNLO re-summation at least two-loop corrections in $\bar{\delta}$ have to be included.

An important question should be raised when one deals with Eqs. (13): How do we define running vev v entering these equations? One option is to assume that $v(\mu)$ is nothing else but the tree-level vev (3) and, thus, gauge-independent. Another common approach utilizes gauge-dependent solution $\phi = \tilde{v}$ of the equation

$$\left. \frac{\partial V_{\text{eff}}(\phi, a(\mu), \mu, \xi)}{\partial \phi} \right|_{\phi=\tilde{v}} = 0. \quad (14)$$

Both approaches have advantages and disadvantages. Let us mention some of them. First of all, particle pole masses are usually identified with the physical ones. Due to this, LHS of (13) are independent of gauge-fixing parameters ξ . If one treats v as the tree-level vev, all ξ -dependence in RHS is explicit. The cancellation is maintained *at the bare level* order-by-order by the inclusion of diagrams with tadpoles attached to *every* particle coupled to the Higgs boson. The drawback of the approach is that the tadpole contributions, which typically scale as powers of $M_t^4/(M_W^2 M_h^2)$, may spoil numerical “convergence” of PT series.

In the “tadpole-free” scheme based on \tilde{v} (14) the running $\overline{\text{MS}}$ masses become implicitly gauge-dependent and it is very hard to check the necessary cancellation in (13). Computations in this case are usually carried out in the Landau gauge $\xi = 0$. The advantage

of this scheme lies in the fact the numerically dangerous tadpole terms are effectively re-summed in \tilde{v} and explicitly contribute only in the Higgs sector (see, e.g., Ref. [13]).

In our study we made use of the first approach. Let us mention, however, that both options should converge to the same result when vev is consistently traded for the Fermi constant G_F by inverting the last relation of (13). This option was advocated, e.g., in Ref. [14] and renders explicitly gauge-independent relations (13) for particle masses.

The set of non-linear equations (13) on the Lagrangian parameters can be solved analytically in PT:

$$\lambda(\mu) = 2^{-1/2} G_F M_H^2 [1 + \delta_H(\mu)], \quad y_f(\mu) = 2^{3/4} G_F^{1/2} M_f [1 + \delta_f(\mu)], \quad (15)$$

where δ_i differ from $\bar{\delta}_i$ and depend on physical masses instead of the running ones.

Due to lack of space, we are not going to present the results for the SM couplings at the EW scale but just refer to our paper [12] together with the C++ code [15]. In spite of the fact that different approaches to tadpoles were utilized in Ref. [16] and our work, numerical results turns out to be very close to each other for the same input taken from PDG2014 [17]. However, our analysis [12] demonstrated that in [16] theoretical uncertainty of the top-quark Yukawa coupling $y_t(M_t)$ seems to be underestimated by a factor of 2.

Let us now return to critical parameters and scales. Simple criterion of (11) serves as a good (and gauge-independent) starting point for our study. A more elaborated condition (7) requires evaluation of V_{eff} . In the latter case the prescription of Ref. [18] was utilized to compute V_{eff} at the two-loop order. According to this reference one not only improves the potential via RG, but also re-organize the expansion (4) by taking into account the scaling $\lambda \sim \hbar g_2^4$. The detailed description of the procedure and the comparison of the simplified and full treatment of V_{eff} can be found in Refs. [12, 18].

A warning should be issued concerning physical meaning of “critical” (or “instability”) scales emerging from the analysis. It is customary to think of these scales as scales, at which some New Physics might appear to cure potential instability. However, one should be careful (see, e.g., discussions [19]) from associating the instability scale with Higgs field values, due to gauge-dependence of the latter. We refrain from doing any conclusion on the New Physics scale and ignore this issue.

The results of vacuum stability analysis are conventionally presented in a form of phase diagram (Fig. 3) in the plane (M_h, M_t) . Two critical lines correspond to the boundaries of the absolute stability region (green) and that of “absolute” instability (red) and are obtained in NNLO. The importance of high-order corrections can be deduced from

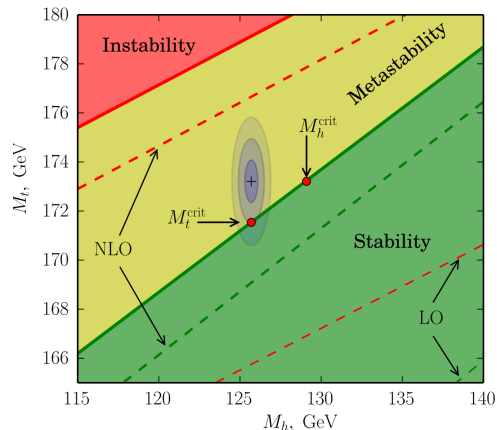


Fig. 3: Phase diagram of vacuum stability obtained at three loops (NNLO). One- (LO) and two-loop (NLO) results are also indicated. The present world average of (M_t, M_h) is shown with 1-3 σ contours.

the comparison with the one-loop (LO) and two-loop (NLO) boundaries. For physical masses lying in the red region (“Instability”) the EW vacuum decays [20] during the age of the Universe τ_U with unit probability. The latter can be estimated from

$$\mathcal{P} \simeq \tau_U \left[\frac{\tau_U^3}{R^4} \exp \left(-\frac{8\pi^2}{3|\lambda(1/R)|} \right) \right], \quad \tau_U \simeq 13.8 \cdot 10^9 \text{ Years} \simeq 6.6 \cdot 10^{41} \text{ GeV}^{-1}. \quad (16)$$

and is less than one in the yellow (“Metastability”) region. In (16) $1/R$ corresponds the scale, at which $(-\lambda)$ is maximized, i.e., $\lambda(1/R) < 0$ and $\beta_\lambda(1/R) = 0$. The derivation of (16) is based on the simple approximation (10) with negative λ and, obviously, require more rigorous, yet gauge-independent, treatment, e.g. along the lines of Ref. [21]. Nevertheless, it turns out that the measured values of M_t and M_h quoted in PDG2014 [17] lie far away from the metastability boundary.

In Fig. 3 we also indicate our results for M_h^{crit} and M_t^{crit}

$$M_t^{\text{crit}} = 171.54 \pm 0.30 \pm 0.41 \text{ GeV}, \quad M_h^{\text{crit}} = 129.1 \pm 1.9 \pm 0.7 \text{ GeV}. \quad (17)$$

The first error in (17) corresponds to parameteric uncertainty due to 1σ variation in the input parameters [17], while the second — to our conservative estimate of theoretical uncertainty coming from unknown high-order terms (see Ref. [12] for details).

To summarize, the SM vacuum issue is studied at the three-loop order. The estimated theoretical uncertainties in critical parameters are comparable with that due to the input, thus, requiring further improvement both from theory and experiment (one anticipates the precision $\delta M_h = 0.04 \text{ GeV}$ and $\delta M_t = 0.1 \text{ GeV}$ at future linear colliders [22]). Due to the fact that the pole mass is ill-defined for color particles, special attention should be paid to the top-quark mass parameter and the interpretation of the value quoted in PDG (see, e.g., [23] and refs therein).¹

It is clear, however, that the whole analysis is carried out at zero temperature and under assumption of the validity of the SM. Obviously, it can be modified by a bunch of factors, including gravity, finite-temperature effects and, finally, New Physics. It is very hard to review all the possibilities so we just refer here to some recent studies [25]. In any case, the EW stability imposes important constraints on possible extensions of the SM. Moreover, due to success of the SM, any New Physics should reproduce it as an effective theory at low energies. And last but not least, various cosmological implications of the analysis can also be studied (see, e.g., [19, 26] and references therein).

Acknowledgments

I am grateful to the organizers for the opportunity to give a talk on the conference dedicated to the 60th anniversary of my home institution - Joint Institute for Nuclear Research. In addition, I would like to thank my colleagues, B. Kniehl, A. Pikelner, O. Veretin and V. Velizhanin, for fruitful collaboration. Inspiring discussions with M. Kalmykov and A. Onischenko are kindly acknowledged. The work is supported in part by Heisenberg-Landau Programme and RFBR grant 14-02-00494-a.

¹Alternative formulation when the stability constraint is imposed on y_t can be found in Ref. [24].

REFERENCES

1. *Hambye T., Riesselmann K.*, Matching conditions and Higgs mass upper bounds revisited // *Phys. Rev.D.* — 1997. — V. 55. — P. 7255–7262.
2. *Coleman S.R., Weinberg E.J.*, Radiative Corrections as the Origin of Spontaneous Symmetry Breaking // *Phys.Rev.D.* — 1973. — V. 7. — P. 1888–1910.
3. *Jackiw R.*, Functional evaluation of the effective potential // *Phys.Rev.D.* — 1974. — V. 9. — P. 1686.
4. *Krasnikov N.V.*, Restriction of the Fermion Mass in Gauge Theories of Weak and Electromagnetic Interactions // *Yad. Fiz.* — 1978. — V. 28. — P. 549–551.
5. *Nielsen N.*, On the Gauge Dependence of Spontaneous Symmetry Breaking in Gauge Theories // *Nucl.Phys.B.* — 1975. — V. 101. — P. 173.
6. *Ford C., Jack I., Jones D.*, The Standard model effective potential at two loops // *Nucl.Phys.B.* — 1992. — V. 387. — P. 373–390.
7. *Froggatt C., Nielsen H.B.*, Standard model criticality prediction: Top mass 173 ± 5 -GeV and Higgs mass 135 ± 9 -GeV // *Phys.Lett.B.* — 1996. — V. 368. — P. 96–102.
8. *Martin S.P.*, Two loop effective potential for a general renormalizable theory and softly broken supersymmetry // *Phys.Rev.D.* — 2002. — V. 65. — P. 116003.
9. *Martin S.P.*, Three-loop Standard Model effective potential at leading order in strong and top Yukawa couplings. — 2013. — arXiv:1310.7553 [hep-ph]; *Martin S.P.*, Four-loop Standard Model effective potential at leading order in QCD // *Phys. Rev.D.* — 2015. — V. 92, no. 5. — P. 054029.
10. *Mihaila L.N., Salomon J., Steinhauser M.*, Gauge Coupling Beta Functions in the Standard Model to Three Loops // *Phys.Rev.Lett.* — 2012. — V. 108. — P. 151602; *Bednyakov A., Pikelner A., Velizhanin V.*, Yukawa coupling beta-functions in the Standard Model at three loops // *Phys.Lett.B.* — 2013. — V. 722. — P. 336–340; *Chetyrkin K., Zoller M.*, β -function for the Higgs self-interaction in the Standard Model at three-loop level // *JHEP.* — 2013. — V. 1304. — P. 091.
11. *Bednyakov A.V., Pikelner A.F.*, Four-loop strong coupling beta-function in the Standard Model. — 2015. — arXiv:1508.02680; *Zoller M.F.*, Top-Yukawa effects on the β -function of the strong coupling in the SM at four-loop level // *JHEP.* — 2016. — V. 02. — P. 095; *Chetyrkin K.G., Zoller M.F.*, Leading QCD-induced four-loop contributions to the β -function of the Higgs self-coupling in the SM and vacuum stability. — 2016. — arXiv:1604.00853.
12. *Bednyakov A.V., Kniehl B.A., Pikelner A.F., Veretin O.L.*, Stability of the Electroweak Vacuum: Gauge Independence and Advanced Precision // *Phys. Rev. Lett.* — 2015. — V. 115, no. 20. — P. 201802.
13. *Actis S., Ferroglia A., Passera M., Passarino G.*, Two-Loop Renormalization in the Standard Model. Part I: Prolegomena // *Nucl. Phys.B.* — 2007. — V. 777. — P. 1–34.

14. *Hempfling R., Kniehl B.A.*, On the relation between the fermion pole mass and MS Yukawa coupling in the standard model // *Phys.Rev.D.* — 1995. — V. 51. — P. 1386–1394; *Kniehl B.A., Veretin O.L.*, Two-loop electroweak threshold corrections to the bottom and top Yukawa couplings // *Nucl.Phys.B.* — 2014. — V. 885. — P. 459.
15. *Kniehl B.A., Pikelner A.F., Veretin O.L.*, mr: a C++ library for the matching and running of the Standard Model parameters. — 2016. — arXiv:1601.08143.
16. *Buttazzo D., Degrandi G., Giardino P.P., Giudice G.F., Sala F., Salvio A., Strumia A.*, Investigating the near-criticality of the Higgs boson // *JHEP.* — 2013. — V. 1312. — P. 089.
17. *Olive K. et al.*, [Particle Data Group Collaboration] Review of Particle Physics // *Chin.Phys.C.* — 2014. — V. 38. — P. 090001.
18. *Andreassen A., Frost W., Schwartz M.D.*, Consistent Use of the Standard Model Effective Potential // *Phys.Rev.Lett.* — 2014. — V. 113, no. 24. — P. 241801.
19. *Espinosa J.R., Giudice G.F., Morgante E., Riotto A., Senatore L., Strumia A., Tetradis N.*, The cosmological Higgstory of the vacuum instability // *JHEP.* — 2015. — V. 09. — P. 174.
20. *Kobzarev I.Yu., Okun L.B., Voloshin M.B.*, Bubbles in Metastable Vacuum // *Sov. J. Nucl. Phys.* — 1975. — V. 20. — P. 644–646. — [*Yad. Fiz.*20,1229(1974)].
21. *Andreassen A., Farhi D., Frost W., Schwartz M.D.*, Precision decay rate calculations in quantum field theory. — 2016. — arXiv:1604.06090.
22. *Arbey A., others.*, Physics at the e+ e- Linear Collider // *Eur. Phys. J.C.* — 2015. — V. 75, no. 8. — P. 371.
23. *Kieseler J., Lipka K., Moch S.O.*, Calibration of the Top-Quark Monte Carlo Mass // *Phys. Rev. Lett.* — 2016. — V. 116, no. 16. — P. 162001.
24. *Bezrukov F., Shaposhnikov M.*, Why should we care about the top quark Yukawa coupling? — 2014.
25. *Eichhorn A., Gies H., Jaeckel J., Plehn T., Scherer M.M., Sondenheimer R.*, The Higgs Mass and the Scale of New Physics // *JHEP.* — 2015. — V. 04. — P. 022; *Burda P., Gregory R., Moss I.*, Gravity and the stability of the Higgs vacuum // *Phys. Rev. Lett.* — 2015. — V. 115. — P. 071303; *Di Luzio L., Isidori G., Ridolfi G.*, Stability of the electroweak ground state in the Standard Model and its extensions // *Phys. Lett.B.* — 2016. — V. 753. — P. 150–160; *Laperashvili L.V., Nielsen H.B., Das C.R.*, New results at LHC confirming the vacuum stability and Multiple Point Principle // *Int. J. Mod. Phys.A.* — 2016. — V. 31, no. 08. — P. 1650029.
26. *Bezrukov F., Rubio J., Shaposhnikov M.*, Living beyond the edge: Higgs inflation and vacuum metastability // *Phys. Rev.D.* — 2015. — V. 92, no. 8. — P. 083512.